

Chapter 1 - Defining a Sturdy element

The set of elements modulo 35 is a set

Working modulo 35 does not give you a group because we can find zero divisors and those are not invertible.

Multiples of 5 and 7 are zero divisors because they are divisors of 35.

There is no inverse for the elements 5,7,10,14,15,20,21,25,28,30 in that set

Listing the elements not on that list gives us a subset consisting of 24 elements as follows
{ 1,2,3,4,6,8,9,11,12,13,16,17,18,19,22,23,24,26,27,29,31,32,33,34 }

Does the zero divisor 5 generate a group?

Powering of 5 just gives us a series of non-invertible results { 5,25,20,30,10,15 }

Our generated results do not include the identity element 1 or similar and as we noted there are no inverses.

Does the element 6 generate a group?

We obtain a cyclic group having 2 elements { 6, 1 }

Does the zero divisor 7 generate a group?

Successive powering of 7 just gives us a series of non-invertible results { 7,14,28,21 }

Our generated results do not include the identity element 1 or similar and as we noted there are no inverses.

Does the element 3 generate a multiplicative group?

We obtain a cyclic group having 12 elements { 3,9,27,11,33,29,17,16,13,4,12,1 }

Does the element 11 generate a multiplicative group?

We obtain a cyclic group having 3 elements { 11,16,1 }

See the next page for sturdy element.

Next we attempt to define what a sturdy element is

- (i) A sturdy element requires context [a set in which it's properties are special]
- (ii) A sturdy element always generates a group [or stronger] from that enclosing context.
- (iii) A sturdy element generates a multiplicative group whose order is $\leq \frac{(n-1)}{2}$
- (iv a) A sturdy element shares the context set with an element that generates a maximal group (n-1) elements (depending on the value of our source variable [t])
- (iv b) A sturdy element shares the context set with an element that generates a near maximal group (n-1-zero divisors based adjustment) elements (depending on the value of our source variable [t])

For the set $1+(-1+2^t) \times (2^{t+1})$ the element $-1+2^t$ is a sturdy element

For the set $1+(-1+2^t) \times (2^{t+1})$ the element $392+t$ is not a sturdy element

Setting $t=5$ we obtain 397 for $392+t$

Working modulo 1985 we obtain the following from successive powering of 397

{ 397, 794, 1588, 1191, 397, ... }

That subset cannot be a group as it contains a zero divisor in 397

That series of elements is not a group so our element $392+t$ fails property (ii) and is therefore not a sturdy element.

For the set $1+(1+2^t) \times (2^{t+1})$ the element $1+2^t$ is a sturdy element

For the set $1+(1+2^t) \times (2^{t+1})$ the element $105+t$ is not a sturdy element

Setting $t=4$ we obtain 109 for $105+t$

Working modulo 545 we obtain the following from successive powering of 109

{ 109, 436, 109 }

That subset cannot be a group as it contains a zero divisor in 109

That series of elements is not a group so our element $105+t$ fails property (ii) and is therefore not a sturdy element.

For the set $1+(-1+3^t)\times(3^{t+1})$ the element $-1+3^t$ is a sturdy element

For the set $1+(-1+3^t)\times(3^{t+1})$ the element $38+t$ is not a sturdy element

Setting $t=5$ we obtain 43 for $38+t$

Working modulo 2107 we obtain the following from successive powering of 43

{ 43, 1849, 1548, 1247, 946, 645, 344, 43, ... }

That subset cannot be a group as it contains a zero divisor in 43

That series of elements is not a group so our element $38+t$ fails property (ii) and is therefore not a sturdy element.

For the set $1+(1+3^t)\times(3^{t+1})$ the element $1+3^t$ is a sturdy element

For the set $1+(1+3^t)\times(3^{t+1})$ the element $25406+t$ is not a sturdy element

Setting $t=5$ we obtain 25411 for $25406+t$

Working modulo 177877 we obtain the following from successive powering of 25411

{ 25411 }

That subset cannot be a group as it contains a zero divisor in 25411

That series of elements is not a group so our element $25406+t$ fails property (ii) and is therefore not a sturdy element.

Examples of a proposed notation for sturdy element are given next.

$\langle\langle -1+2^t \rangle\rangle$ for set $1+(-1+2^t)\times(2^{t+1})$ for $t=4,5$

$\langle\langle 1+2^t \rangle\rangle$ for set $1+(1+2^t)\times(2^{t+1})$ for $t=4,5$

$\langle\langle -1+3^t \rangle\rangle$ for set $1+(-1+3^t)\times(3^{t+1})$ for $t=4,5$

$\langle\langle 1+3^t \rangle\rangle$ for set $1+(1+3^t)\times(3^{t+1})$ for $t=4,5$